

## Exercise 52

Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{2e^x}{e^x - 5}$$

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### Solution

Calculate the limits as  $x \rightarrow \pm\infty$  to determine the horizontal asymptote. In the second limit, make the substitution,  $x = -u$ , so that as  $x \rightarrow -\infty$ ,  $u \rightarrow \infty$ .

$$\lim_{x \rightarrow \infty} \frac{2e^x}{e^x - 5} = \lim_{x \rightarrow \infty} \frac{2}{1 - \frac{5}{e^x}} = \frac{2}{1 - 0} = 2$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{2e^x}{e^x - 5} &= \lim_{u \rightarrow \infty} \frac{2e^{-u}}{e^{-u} - 5} \\ &= \lim_{u \rightarrow \infty} \frac{\frac{2}{e^u}}{\frac{1}{e^u} - 5} \\ &= \frac{0}{0 - 5} \\ &= 0 \end{aligned}$$

Therefore, the horizontal asymptotes are  $y = 2$  and  $y = 0$ . The vertical asymptotes are found by setting what's in the denominator equal to zero and solving for  $x$ .

$$e^x - 5 = 0$$

$$e^x = 5$$

$$\ln e^x = \ln 5$$

$$x \ln e = \ln 5$$

$$x(1) = \ln 5$$

$$x = \ln 5$$

The function is graphed versus  $x$  below with the asymptotes labelled.

